ENSAE Paris - IP Paris
Academic year 2025-2026
First exam session (2A S1)

Final exam of the course "Macroeconomics 1"

Duration : 2 hours. No document allowed, no calculator allowed.
The grading scale, which may be modified, is indicated only for general-guidance purposes.

1 Lecture questions (6 points)

Indicate the correct answer (a, b, c or d), without giving any explanation. There is only one correct answer. For each question, the correct answer gives 1 point, no answer or several answers give 0 point, and **an incorrect answer removes 0.25 point**.

Question 1 An exogenous-growth theory is a theory in which the long-term growth rate is equal to an
a. exogenous saving rate,
b. endogenous saving rate,
c. exogenous rate of technical progress,
d. endogenous rate of technical progress.

Question 2 There is dynamic inefficiency when
a. per-capita consumption could be raised in the short term and in the long term,
b. per-capita production could be raised in the short term and in the long term,
c. per-capita consumption is higher in the short term than in the long term,
d. per-capita production is higher in the short term than in the long term.

Question 3 The statement "In the DICE model at the steady state, everything else equal, the higher the growth rate of the economy (g), the [X] the marginal utility of future consumption relatively to the marginal utility of current consumption, and the [Y] the discount rate (r)" is true if
a. X = lower, Y = lower,
b. X = lower, Y = higher,
c. X = higher, Y = lower,
d. X = higher, Y = higher.

Question 4 In the growth model with learning by doing (Romer, 1986), the knowledge-diffusion externality
a. prevents the planner's allocation from being socially optimal,

b. leads to too low an initial consumption level in the competitive equilibrium under laisser-faire,
c. can be corrected by a Pigouvian tax on investment,
d. can be corrected by a Pigouvian subsidy on financial incomes.

Question 5 In the growth model with product variety (Romer, 1990), the social sub-optimality of the market equilibrium under laisser-faire
a. is due to the research-and-development (R&D) externality,
b. can be corrected by a subsidy to R&D,
c. is due to monopolistic competition, and the latter encourages innovation,
d. is due to monopolistic competition, and the latter discourages innovation.

Question 6 The Ricardian equivalence states that
a. the effect of public debt on the economy does not depend on the amount of taxes,
b. the effect of taxes on the economy does not depend on the amount of public expenditures,
c. the effect of public expenditures on the economy does not depend on the way they are financed,
d. the way public expenditures are financed does not depend on the amount of public debt.

2 Problem : CES production function in the Cass-Koopmans-Ramsey model (14 points)

The goal of this problem is to study the positive consequences of a change in the production function in the Cass-Koopmans-Ramsey model. **One can answer each question without having answered the previous questions**, simply by using the results given in these previous questions. **The questions with an asterisk (*) will be worth a bit more points than the other questions.**

We start from the Cass-Koopmans-Ramsey model considered in the lectures and the tutorials, in the particular case of a constant elasticity of intertemporal substitution, equal to $1/\theta$, where $\theta > 0$. We use exactly the same notations as in the lectures and the tutorials : r_t denotes the real interest rate, w_t the real wage, b_t the per-capita real amount of assets, L_t the population and aggregate labor supply, C_t aggregate consumption, $c_t \equiv C_t/L_t$ per-capita consumption, K_t the aggregate capital stock, $k_t \equiv K_t/L_t$ the per-capita capital stock, Y_t aggregate output, and $y_t \equiv Y_t/L_t$ per-capita output at time t ; $\rho > 0$ denotes the rate of time preference, $n \geq 0$ the population-growth rate, and $\delta > 0$ the capital-depreciation rate.

The only two differences from the Cass-Koopmans-Ramsey model considered in the lectures and the tutorials are the following : 1) we assume here that there is no technical progress ($g = 0$, so labor effectiveness A_t is a constant that we normalize to one), and 2) we consider here a production function F potentially different from the one of the Cass-Koopmans-Ramsey model considered in the lectures and the tutorials. We note $Y_{i,t} = F(K_{i,t}, N_{i,t})$, where $Y_{i,t}$, $K_{i,t}$, $N_{i,t}$ respectively denote the output, the capital stock, and the labor demand of firm i at time t, and we note $f(x) \equiv F(x, 1)$. Finally, as in the lectures and the tutorials, we focus on parameter values such that $\rho - n > (1 - \theta)g$, that is to say here such that $\rho > n$ (since $g = 0$).

2.1 Generic production function

We first consider a production function F that is strictly increasing and strictly concave in each of its arguments, and homogeneous of degree one.

Question 7* It is reminded that, in this model, the representative household's optimization problem is the following one : for some given $(r_t, w_t)_{t\geq 0}$ and b_0,

$$\max_{(c_t)_{t\geq 0},(b_t)_{t>0}} \int_0^{+\infty} e^{-(\rho-n)t} \left(\frac{c_t^{1-\theta}-1}{1-\theta} \right) dt$$

subject to

$$\forall t \geq 0,\ c_t \geq 0,$$
$$\forall t \geq 0,\ \dot{b_t} = (r_t - n)b_t + w_t - c_t,$$
$$\lim_{t\to+\infty} \left[b_t e^{-\int_0^t (r_\tau - n)d\tau} \right] \geq 0.$$

Briefly interpret, in economic terms, the last two constraints. Write the Hamiltonian of the representative household's problem ; write the first-order condition on the control variable and the costate equation, and then get the Euler equation. Briefly explain why we have $r_t = f'(k_t) - \delta$ in equilibrium ; deduce the following differential equation :

$$\frac{\dot{c_t}}{c_t} = \frac{1}{\theta} \left[f'(k_t) - (\delta + \rho) \right]. \tag{1}$$

Briefly interpret, in economic terms, the way in which $\dot{c_t}/c_t$ depends on δ and ρ.

Question 8 Write the goods-market-clearing condition in aggregate terms. Deduce the following differential equation :

$$\dot{k_t} = f(k_t) - c_t - (n + \delta)\, k_t. \tag{2}$$

2.2 CES production function

From now on, we consider the following production function (called "constant-elasticity-of-substitution function" or "CES function") :

$$F(K_{i,t}, N_{i,t}) \equiv \Lambda \left[\alpha K_{i,t}^{\psi} + (1-\alpha)\, N_{i,t}^{\psi} \right]^{\frac{1}{\psi}},$$

where $\Lambda > 0$, $0 < \alpha < 1$, $0 < \psi < 1$. We thus have $f(x) \equiv F(x, 1) = \Lambda[\alpha x^{\psi} + (1-\alpha)]^{1/\psi}$.

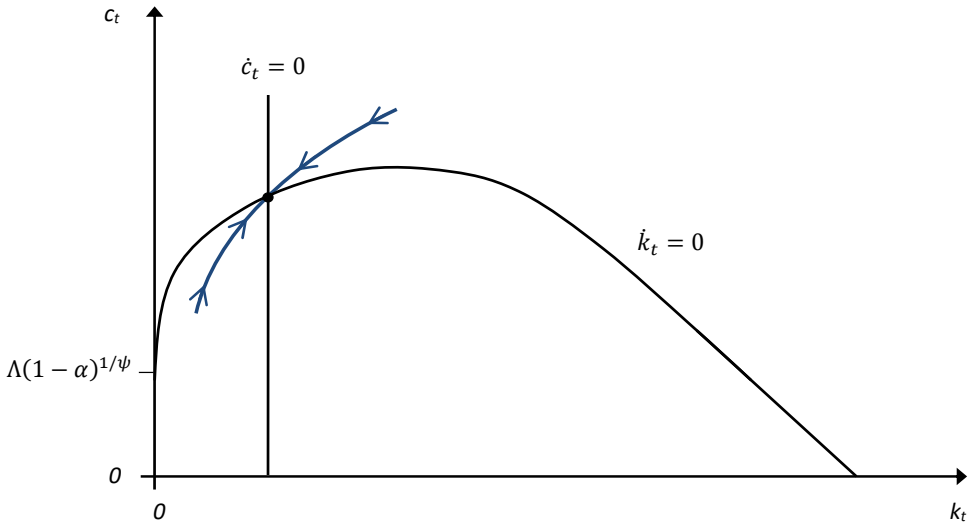
Question 9 Of the five properties of the Cass-Koopmans-Ramsey model's production function (considered in the lectures and the tutorials), which one(s) does the CES production function not satisfy ? (For the following three properties : strict increasingness, strict concavity, Inada conditions, please focus on the one-variable function f, rather than studying the two-variable function F.) Deduce in particular that the differential equations (1) and (2) still hold for this production function.

2.3 Case $A < n + \delta$

Let $A \equiv \Lambda\alpha^{1/\psi}$. In this section, we consider the case $A < n + \delta$, which implies in particular $A < \delta + \rho$ (since $\rho > n$).

Question 10 Explain why the equations $\dot{k_t} = 0$ and $\dot{c_t} = 0$ correspond to a bell-shaped curve and a vertical straight line in the quadrant $(k_t > 0, c_t > 0)$ of the plane (k_t, c_t), as in the Cass-Koopmans-Ramsey model considered in the lectures and the tutorials, and as represented in the figure below. Assume that, in the absence of shocks and surprises,

the unique equilibrium path is the saddle path, as in the Cass-Koopmans-Ramsey model considered in the lectures and the tutorials, and as represented in the figure below. [1] Deduce $\lim_{t\to+\infty}(\dot{y}_t/y_t)$. Interpret. Is the model an exogenous-growth or an endogenous-growth model ?



2.4 Case $A > \delta + \rho$ and $\varphi > 0$

Let $\varphi \equiv (A-\delta)(\theta-1)/\theta + \rho/\theta - n$. In this section, we consider the case $A > \delta + \rho$ and $\varphi > 0$.

Question 11 Let $\chi_t \equiv c_t/k_t$ denote the consumption-capital ratio, and $z_t \equiv y_t/k_t$ the output-capital ratio (also equal to the average productivity of capital : $z_t = f(k_t)/k_t$). Using $\dot{z}_t/z_t = [f'(k_t)k_t/f(k_t) - 1]\dot{k}_t/k_t$ and (2), show that

$$\frac{\dot{z}_t}{z_t} = \left[\left(\frac{A}{z_t}\right)^{\psi} - 1\right][z_t - \chi_t - (n+\delta)]\,. \tag{3}$$

Using $\dot{\chi}_t/\chi_t = \dot{c}_t/c_t - \dot{k}_t/k_t$, (1) and (2), show that

$$\frac{\dot{\chi}_t}{\chi_t} = \frac{A}{\theta}\left[\left(\frac{z_t}{A}\right)^{1-\psi} - 1\right] - (z_t - A) + (\chi_t - \varphi)\,. \tag{4}$$

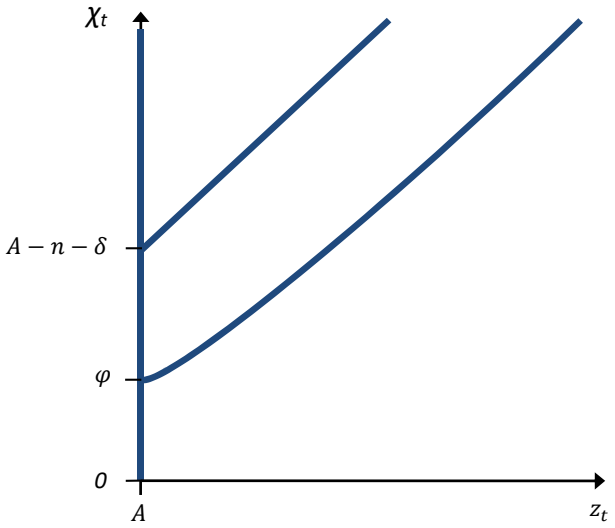
Question 12 In this specific context, the steady state is defined as a situation in which z_t and χ_t grow at some constant rates, respectively denoted by μ_z and μ_χ. Show that $z_t \geq A$; deduce that $\mu_z \geq 0$. In the hypothetical case in which $\mu_z > 0$, determine $\lim_{z_t\to+\infty}(z_t - \chi_t)$ at the steady state according to (3) on the one hand and to (4) on the other hand ; deduce that $\mu_z = 0$, and then that $\mu_\chi = 0$. Assuming that there exists a unique steady state, show that $z_t = A$ and $\chi_t = \varphi$ at the steady state.

Question 13* Assume that the economy converges over time to the steady state. Deduce $\lim_{t\to+\infty} z_t$, $\lim_{t\to+\infty} k_t$, and $\lim_{t\to+\infty}(\dot{y}_t/y_t)$. Briefly interpret the way in which this last limit depends on A, δ and ρ. Is the model an exogenous-growth or an endogenous-growth model ?

Question 14* Reproduce the figure below on your exam paper. In this figure, two straight (half-)lines and one curve are drawn in bold ; indicate what each of them represents (either

1. The proof of this result is essentially the same as the one seen in the lectures.

a set of points such that $\dot{z}_t = 0$, or a set of points such that $\dot{\chi}_t = 0$). Then indicate the steady-state point, and check that $\varphi < A - n - \delta$. Using horizontal and vertical arrows, indicate whether z_t and χ_t increase or decrease over time in each of the three zones that are bounded by these straight (half-)lines and this curve. Deduce the shape of the saddle path that leads to the steady state. Interpret the way in which z_t and χ_t vary over time along this saddle path.



Question 15* Assume that the economy moves along the saddle path until time $T > 0$ (excluded). At time T, unexpectedly, the parameter Λ takes a new value, higher than its previous value, and remains at this new value thereafter. Draw the path followed by the economy in the plane (z_t, χ_t), in the specific case in which $\theta = 1$. Explain and interpret.