New Principles For Stabilization Policy

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Determinacy conditions in macroeconomics

- Dynamic **rational-expectations models** widely used in macroeconomics.
- Natural goal for **stabilization policy** in these models: ensure "**determinacy**" (i.e. a unique local equilibrium), to avoid undesirable macroeconomic fluctuations.
- Large theoretical and empirical literature about **conditions** on the coefficients of the policy-instrument rule to get determinacy.
- Best known result: "**Taylor principle**" for monetary policy.

Limitation of the literature

- However, **no general picture and no good understanding** of det. conditions:
  - det. conditions studied only on a model-by-model, rule-by-rule basis,
  - analytical det. conditions obtained only in (very) simple contexts,
  - Taylor principle sometimes not nec. or not suff. for determinacy.
- **Main difficulty** in getting general results:
  - Blanchard and Kahn's (1980) det. conditions are about polynomial roots,
  - these roots depend on the policy-instrument rule in a complicated way.
- In this paper, I use two complex-analysis theorems to overcome this difficulty.

Contribution of the paper

- I consider a broad class of discrete-time rational-expectations models, and the class of (locally log-linearized) policy-instrument rules of type

$$\rho(L) i_t = \phi \mathbb{E}_t \{v_{t+h}\} + \sum_{j=1}^{J} \phi_j \mathbb{E}_t \{v_{j,t+h_j}\} .$$

- I establish analytically some **simple, easily interpretable, necessary or sufficient conditions for determinacy** on the coefficient $\phi \in \mathbb{R}$ and the horizon $h \in \mathbb{Z}$.

- These conditions lead to **new principles for stabilization policy** in terms of whether, and how strongly or weakly, to react to any variable, at any horizon, in any model, with any policy instrument.

- I characterize the scope of validity of the (generalized) long-run **Taylor principle** as a condition for determinacy.

- I **apply** all these results to standard interest-rate rules in 134 quantitative monetary-policy models.

Related Literature

- **Analytical determinacy conditions**: Benhabib et al. (2001), Bullard and Mitra (2002), Carlstrom and Fuerst (2002), Woodford (2003), ..., Acharya and Dogra (2020), Gabaix (2020), Bilbiie (2024).
- **Complex-analysis theorems**: Rouché (1862), Erdős and Turán (1950); Bhattarai et al. (2014).
- **Horizon of the rule and (in)determinacy**: Levin et al. (2003), Benhabib (2004), Loisel (2024).
- **US economy and (in)determinacy**: Clarida et al. (2000), Lubik and Schorfheide (2004), Beaudry et al. (2017, 2020).
- **Fiscal policy and (in)determinacy**: Leeper (1991), Schmitt-Grohé and Uribe (1997).
- **Robustness of interest-rate rules**: Levin et al. (1999, 2003), Levin and Williams (2003), Taylor and Williams (2011), Wieland et al. (2012, 2016), Holden (2024).

Outline

1. A basic New Keynesian illustration

2. General analysis

3. Quantitative application to monetary policy

Model and rule

- **Structural equations**:

$$
\begin{aligned}
y_t &= \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\{\pi_{t+1}\}\right), \\
\pi_t &= \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa y_t.
\end{aligned}
$$

- **Interest-rate rule**:

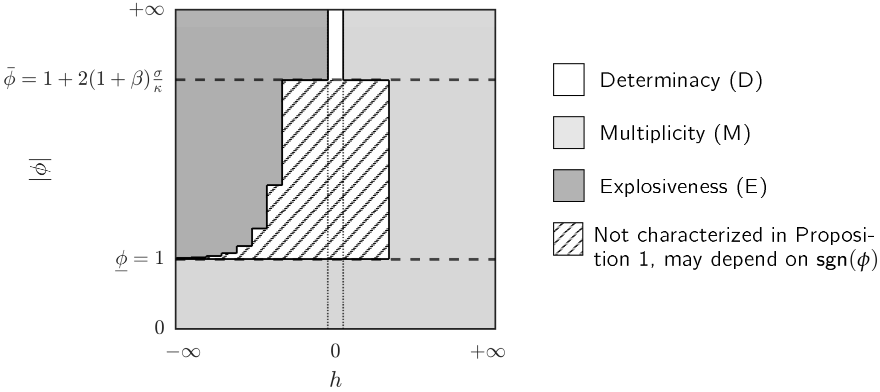
$$
i_t = \phi \mathbb{E}_t\left\{\pi_{t+h}\right\}. \qquad \text{(Rule 1)}
$$

- Resulting **dynamic equation**:

$$
\mathbb{E}_t\left\{\beta\pi_{t+2} - \left(1+\beta+\frac{\kappa}{\sigma}\right)\pi_{t+1} + \pi_t + \phi\frac{\kappa}{\sigma}\pi_{t+h}\right\} = 0.
$$

- Let $S(\phi, h) \in \{M, D, E\}$ denote the "**determinacy status**" (M for "multiplicity", D for "determinacy", E for "explosiveness").

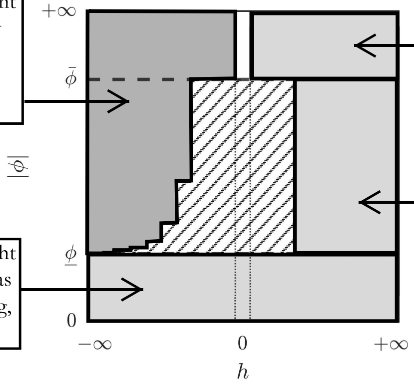
Prop. 1: $S(\phi, h)$, independently of $\operatorname{sgn}\phi$



Basic intuitions for Prop. 1

A sufficiently **large** weight on inflation sufficiently distant in the **past** favors exploding paths and leads to **E**.

A sufficiently **small** weight on inflation leads to **M**, as under an interest-rate peg, by continuity.

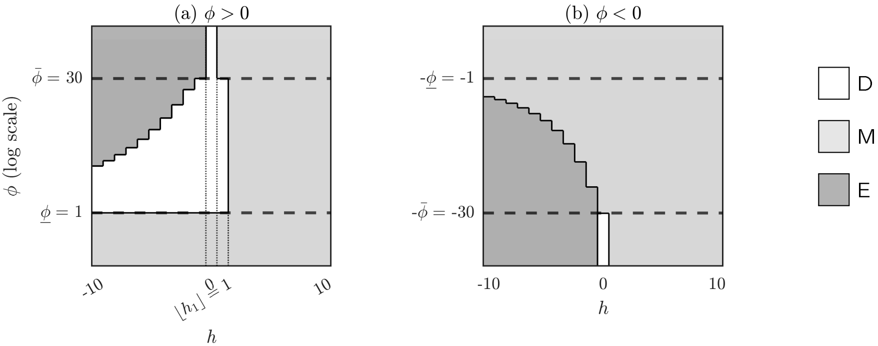


A sufficiently **large** weight on inflation sufficiently distant in the **future** favors imploding paths and leads to **M**.

A sufficiently **small** weight on inflation sufficiently distant in the **future** leads to **M**, as under an interest-rate peg, by continuity.

$S(\phi, h)$, depending on $\operatorname{sgn}\phi$

- $S(\phi, h)$ for Woodford's (2003) calibration of the basic NK model:



- Prop. 2: $\forall \phi \in (-\bar{\phi}, -\underline{\phi})$, $\forall h \in \mathbb{Z}$, $S(\phi, h) \neq D$.

Taylor principle

- Prop. 3: *For $\phi > 0$, the* **Taylor principle** $\phi > 1$ *is necessary and locally sufficient for determinacy if and only if $h < h_1 := 1 + (1 - \beta)\sigma/\kappa$.*
- **Intuition**:
  - for $\phi \in (0, 1)$, we are missing one characteristic-polynomial root outside the unit circle $\mathcal{C}$ to get determinacy (just like under a peg);
  - as ϕ goes from below 1 to above 1, one root crosses the unit circle $\mathcal{C}$;
  - when $\mathbf{h} < \mathbf{h_1}$, the root goes from inside to outside $\mathcal{C}$ (increasing the weight on inflation sufficiently distant in the **past** favors **exploding** paths);
  - when $\mathbf{h} > \mathbf{h_1}$, the root goes from outside to inside $\mathcal{C}$ (increasing the weight on inflation sufficiently distant in the **future** favors **imploding** paths).

Rule inertia I

- **Inertial rule**:
$$i_t = \rho i_{t-1} + (1-\rho)\,\phi\mathbb{E}_t\{\pi_{t+h}\}, \qquad \text{(Rule 2)}$$
where $\rho \in (0,1)$.

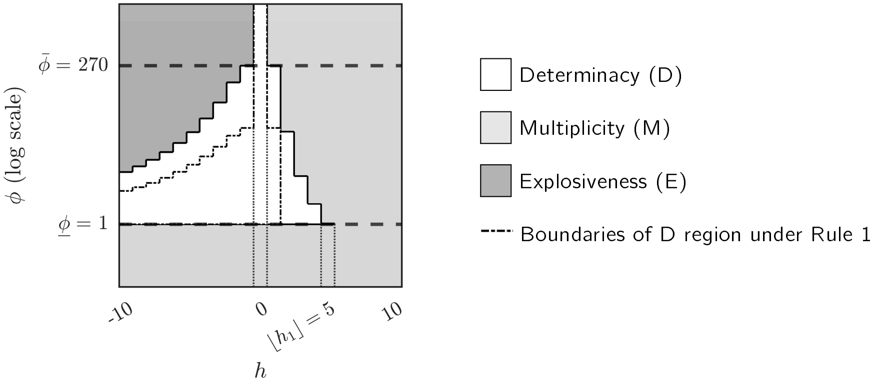
- Prop. 4: *Propositions 1-3 still hold for Rule 2 instead of Rule 1, with $\underline{\phi}$ unchanged, $\bar{\phi}$ multiplied by $(1+\rho)/(1-\rho)$, and* $\mathbf{h_1}$ **increased by** $\boldsymbol{\rho/(1-\rho)}$.

- **Intuition** for the increase in h_1: inertia, by increasing the weight on **past** outcomes, tends to favor **exploding** paths.

- Note that h_1 increases **unboundedly** as $\rho \to 1$.

Rule inertia II

- $S(\phi, h)$ for Woodford's (2003) calibration, $\rho = 0.8$ and $\phi > 0$:



Model and rule

- **Structural equations**:

$$\mathbb{E}_t \left\{ \underset{(n\times n)}{\boldsymbol{\Delta}(L^{-1})} \left[\underset{(n\times n)}{\mathbf{A}(L)} \underset{(n\times 1)}{\mathbf{X}_t} + L^{-\gamma} \underset{(n\times 1)}{\mathbf{B}(L)} i_t \right] \right\} = \mathbf{0}.$$

- **Policy-instrument rule**:

$$i_t = \phi \mathbb{E}_t \left\{ v_{t+h} \right\},$$

where $v_t := \underset{(1\times n)}{\mathbf{V}(L)} \underset{(n\times 1)}{\mathbf{X}_t}$.

- Reciprocal polynomial of the **characteristic polynomial** (RPCP):

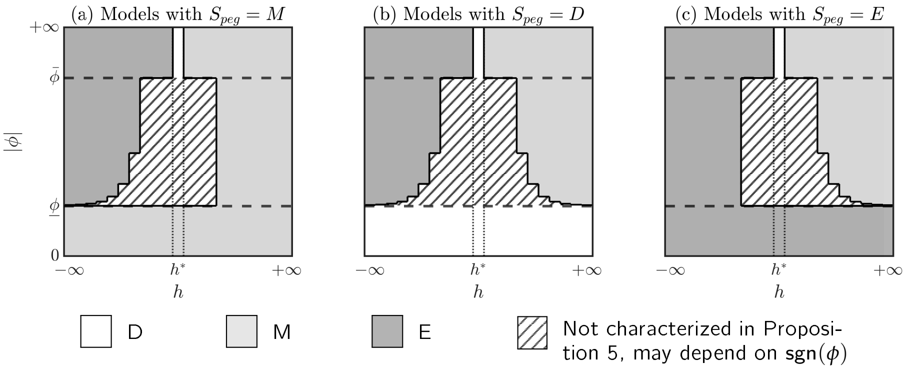
$$P(z) = \underbrace{Q(z)}_{\substack{\text{RPCP under} \\ \text{a peg } (i_t = 0)}} z^{\max(0,h-m)} + \phi \underbrace{R(z)}_{\substack{\text{RPCP under} \\ \text{the "targeting} \\ \text{rule" } v_t = 0}} z^{\max(0,m-h)}.$$

Under a peg

- Let $\mathbf{S_{peg}} \in \{M, D, E\}$ denote the **determinacy status under a peg** ($\phi = 0$).
- Let $\mathbf{d_{peg}} \in \mathbb{Z}$ denote the **degree of indeterminacy under a peg**.
- E.g., for five simple calibrated monetary-policy models:

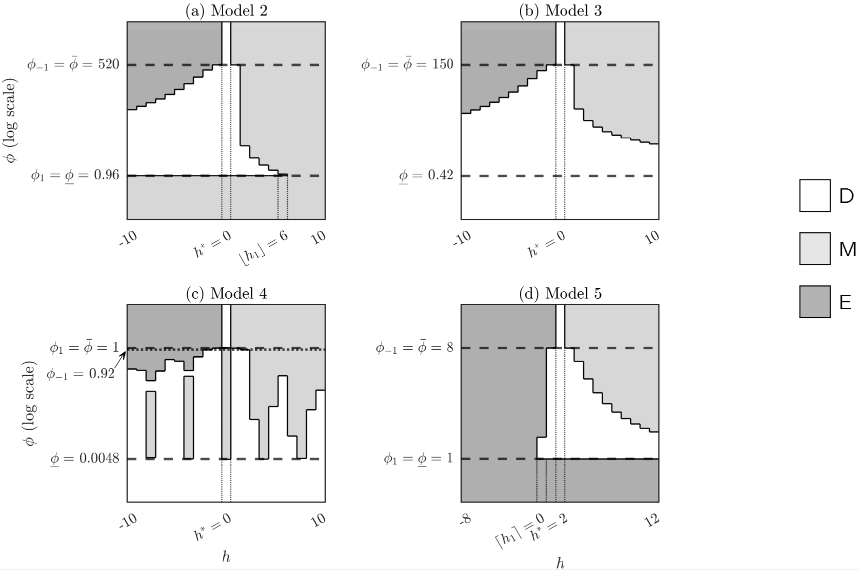
No.	**Model**	**Calibration**	$\mathbf{S_{peg}}$	$\mathbf{d_{peg}}$
1	Basic NK Model	Woodford (2003)	M	1
2	McKay et al. (2017)	McKay et al. (2017)	M	1
3	Gabaix (2020)	Gabaix (2020)	D	0
4	Bilbiie (2008)	Bilbiie (2008)	D	0
5	Svensson (1997) and Ball (1999)	Ball (1999)	E	-1

Prop. 5: $S(\phi, h)$, independently of sgn ϕ



where $\underline{\phi} := \min_{z \in \mathcal{C}} \left| \frac{Q(z)}{R(z)} \right|$ and $\bar{\phi} := \max_{z \in \mathcal{C}} \left| \frac{Q(z)}{R(z)} \right|$.

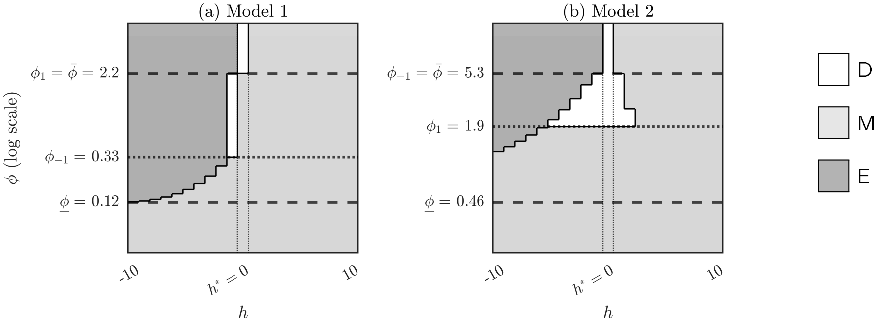
$S(\phi, h)$ for Models 2-5 and Rule 1 with $\phi > 0$



$S(\phi, h)$ for some ϕ intervals

- Prop. 6 identifies some intervals of ϕ values, inside $(\underline{\phi}, \bar{\phi})$ and $(-\bar{\phi}, -\underline{\phi})$, for which $S(\phi, h) \neq D$ for all $h \in \mathbb{Z}$.

- E.g., for Models 1-2 and the rule $i_t = \phi \mathbb{E}_t \{y_{t+h}\}$ with $\phi > 0$:



$$\text{where} \quad \phi_1 := -Q(1)/R(1) \quad \text{and} \quad \phi_{-1} := |Q(-1)/R(-1)|.$$

Taylor principle

- Definition (long-run **Taylor principle** – generalization of Woodford, 2003):
  If $\phi_1 > 0$, then the Taylor principle (TP) is $\phi > \phi_1$.

- (Extract from) Prop. 7: *Let $h_1 := m + R'(1)/R(1) - Q'(1)/Q(1)$.*

  - *If $\boldsymbol{\phi_1 = \bar{\phi}}$, then the TP is sufficient for E (if $h \leq h^* - 1$), or D (if $h = h^*$), or M (if $h \geq h^* + 1$).*

  - *If $\boldsymbol{\phi_1 \in (\underline{\phi}, \bar{\phi})}$, then the TP is locally suff. for D for finitely many or no h's.*

  - *If $\boldsymbol{\phi_1 = \underline{\phi}}$, then for $\phi > 0$, the TP is necessary and locally sufficient for D if and only if ($d_{peg} = 1$ and $h < h_1$) or ($d_{peg} = -1$ and $h > h_1$).*

- The distinction $\phi_1 = \underline{\phi}$ vs. $\phi_1 = \bar{\phi}$ sheds light on some contrasting results about the Taylor principle in the monetary-policy literature (e.g., Bilbiie, 2008).

Rule inertia

- **Inertial rule**:

$$\rho(L) i_t = \rho(1) \phi \mathbb{E}_t \left\{ v_{t+h} \right\},$$

  where $\rho(z) \in \mathbb{R}[z]$ with $\rho(0) \neq 0$.

- In this presentation, I focus on non-superinertial rules (i.e. rules such that $\rho(z)$ has no roots inside $\mathcal{C}$); but I allow for superinertial rules in the paper.

- (Extract from) Prop. 8: *Propositions 5-7 still hold for the inertial rule instead of the non-inertial rule, with h^* and ϕ_1 unchanged, and* $\boldsymbol{-\rho'(1)/\rho(1)}$ **added to** $\mathbf{h_1}$.

- So, for $\rho(z) = 1 - \rho z$ (as in Rule 2), $\mathbf{h_1}$ **increases by** $\boldsymbol{\rho/(1-\rho)}$, just like in the basic NK illustration.

Rules with several variables

- **Rule with several variables**:

$$\rho(L) i_t = \rho(1) \left(\phi \mathbb{E}_t \left\{ v_{t+h} \right\} + \sum_{j=1}^{J} \phi_j \mathbb{E}_t \left\{ v_{j,t+h_j} \right\} \right).$$

- Rewrite this rule as a "structural equation" combined with a single-variable rule:

$$\begin{aligned} \rho(L) i_t &= \rho(1) \left(\tilde{i}_t + \sum_{j=1}^{J} \phi_j \mathbb{E}_t \left\{ v_{j,t+h_j} \right\} \right), \\ \tilde{i}_t &= \phi \mathbb{E}_t \left\{ v_{t+h} \right\}, \end{aligned}$$

and then apply Propositions 5-7 to the modified model and the single-variable rule.

MMB models

- I apply the general results to standard interest-rate rules in **134 quantitative monetary-policy models**.
- The 134 models belong to the 140 rational-expectations models of the Macroeconomic Model Data Base (MMB) described in Wieland et al. (2012, 2016).
- MMB models differ in various dimensions (size, microfoundations, rigidities, frictions, openness, agents, data, policymaking, etc).
- Distribution of d_{peg} across MMB models *(140 models)*:

Value of d_{peg}	-1	0	1
Number of models	6	4	130

Rules

- **Six standard interest-rate rules**:

$$i_t = \phi \mathbb{E}_t \{\pi_{t+h}\}, \tag{Rule 1}$$

$$i_t = \rho i_{t-1} + (1-\rho)\, \phi \mathbb{E}_t \{\pi_{t+h}\}, \tag{Rule 2}$$

$$i_t = \phi \mathbb{E}_t \{\pi_{t+h} + (1/3) y_{t+h}\}, \tag{Rule 3}$$

$$i_t = \phi \mathbb{E}_t \{\pi_{t+h}\} + (1/2) y_t, \tag{Rule 4}$$

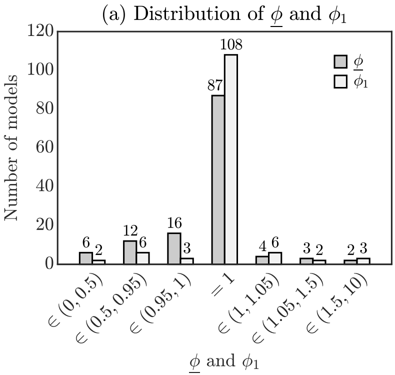
$$i_t = \rho i_{t-1} + (1-\rho)\, \phi \mathbb{E}_t \{\pi_{t+h} + (1/3) y_{t+h}\}, \tag{Rule 5}$$

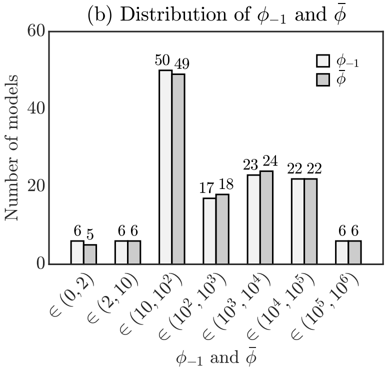
$$i_t = \rho i_{t-1} + (1-\rho)\, [\phi \mathbb{E}_t \{\pi_{t+h}\} + (1/2) y_t], \tag{Rule 6}$$

where $\rho = 0.8$.

- For $(\phi, h) = (1.5, 0)$, Rules 3 and 4 coincide with each other and take the familiar form $i_t = 1.5\pi_t + 0.5 y_t$.

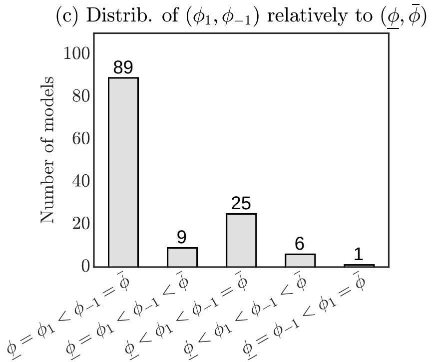
Distributions under Rule 1 *(130 models)* I

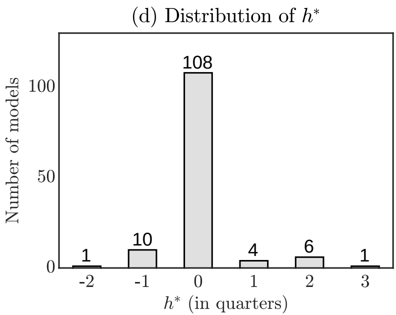




- $\underline{\phi}$ and ϕ_1 typically equal or close to 1, ϕ_{-1} and $\bar{\phi}$ typically one or several orders of magnitude larger (although in $(0, 2)$ for a few models), like in basic NK illustration.
- Similar results for Rules 2-6 (in particular, ϕ_1 remains close to 1 under Rules 3-4 because long-run Phillips curves are approximately vertical).

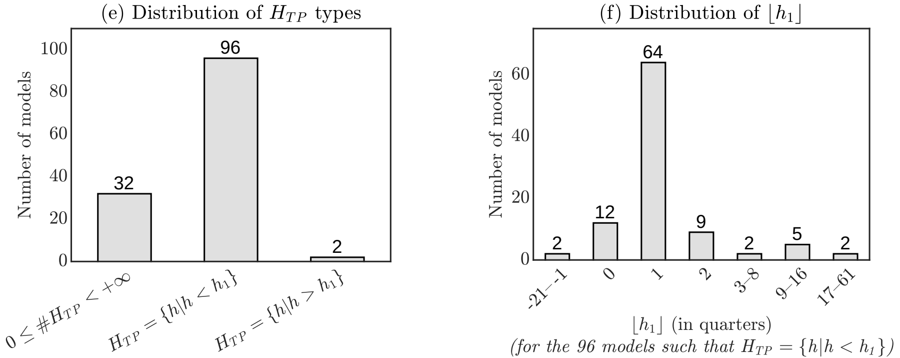
Distributions under Rule 1 *(130 models)* II





- For most models, $(\phi_1, \phi_{-1}) = (\underline{\phi}, \bar{\phi})$ and $h^* = 0$, like in basic NK illustration.
- Similar results for Rules 2-6.

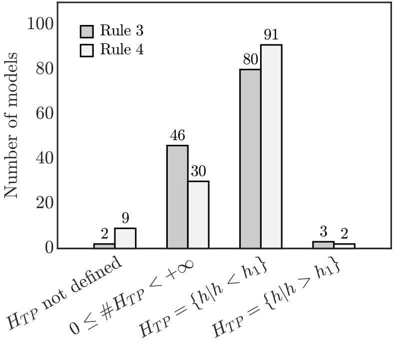
Distributions under Rule 1 *(130 models)* III



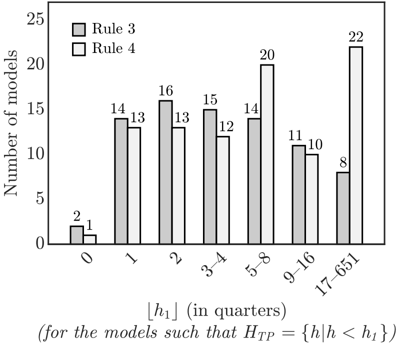
- For most models, $H_{TP} := \{h|\textit{the TP is necessary and locally sufficient for } D\}$ of type $\{h|h < h_1\}$ and $\lfloor h_1 \rfloor = 1$, like in basic NK illustration.
- Under Rule 2, H_{TP} still predominantly of type $\{h|h < h_1\}$, but $\lfloor h_1 \rfloor$ increases by 4 quarters (typically from 1 to 5 quarters).

Distributions under Rules 3-4 *(131-132 models)*

(a) Distribution of H_{TP} types



(b) Distribution of $\lfloor h_1 \rfloor$



- H_{TP} still predominantly of type $\{h|h < h_1\}$, but $\lfloor h_1 \rfloor$ larger than under Rule 1.
- Under Rules 5-6, H_{TP} still predominantly of type $\{h|h < h_1\}$, but $\lfloor h_1 \rfloor$ even larger (essentially by 4 quarters).

Robust rules

- The application shows that the new principles for stabilization policy can be **quantitatively relevant**.
- The application also provides guidelines for finding a **robust** interest-rate rule.
- Using five models and a grid of rule-coefficient values, Levin et al. (2003) identified four characteristics of interest-rate rules that deliver determinacy:
  - “a relatively short inflation forecast horizon,”
  - “a moderate degree of responsiveness to the inflation forecast,”
  - “a substantial degree of policy inertia,”
  - “an explicit response to the current output gap.”
- The application shows that these four characteristics actually favor determinacy in **most** MMB models, and explains **why** they do.

Conclusion

- The paper has established some simple, general, necessary or sufficient **conditions for determinacy** in a broad class of models.
- These determinacy conditions are directly about the coefficients and horizons of the policy-instrument rule, and lead to **new principles for stabilization policy**.
- These conditions also characterize the scope of validity of the (generalized) long-run **Taylor principle** as a condition for determinacy.
- The paper has applied all these results to standard interest-rate rules in 134 quantitative monetary-policy models.
- More generally, the results can be applied to any stabilization policy (unconventional monetary policy, fiscal policy, macroprudential policy...).